Physics of staircases in EM turbulence -- A simple model

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Outline

- Background
- Fixed Cellular Array (FCA) Problem
- Relaxing FCA w/ Fluctuating Vortex Array (FVA)
- Active Scalar Dynamics
- Active Scalar Summary

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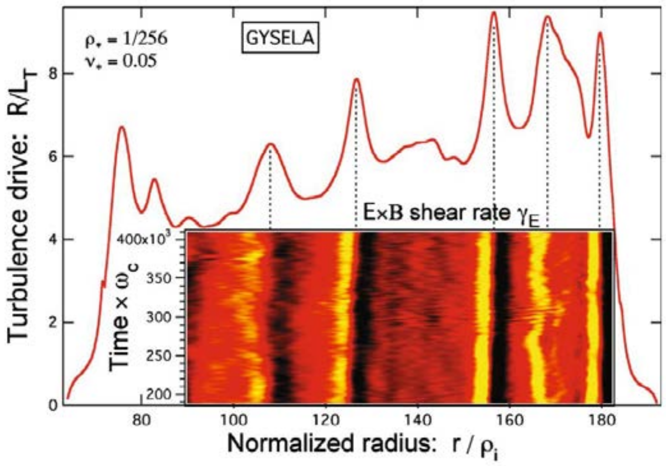
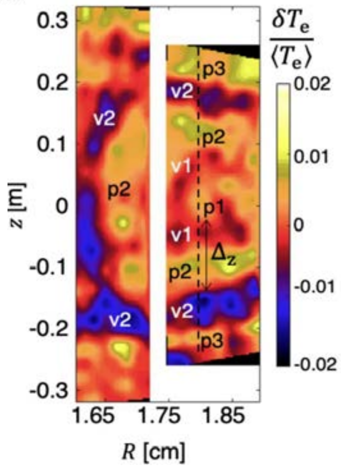
Background

E×B Staircase

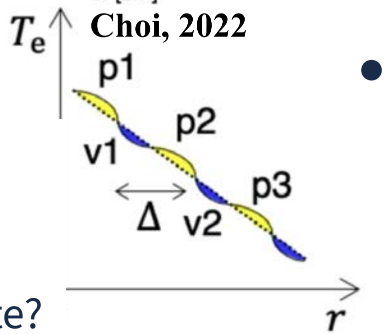
Dif-Pradalier, 2017



KSTAR



Choi, 2022



Context: Flat spots of high transport and nearly vertical layers acting as mini-barriers coexist. In plasmas, avalanches happen in flat spots and shear layers due to zonal flows occur in the areas of mini-barriers

Suggested ideas (from self-organization):

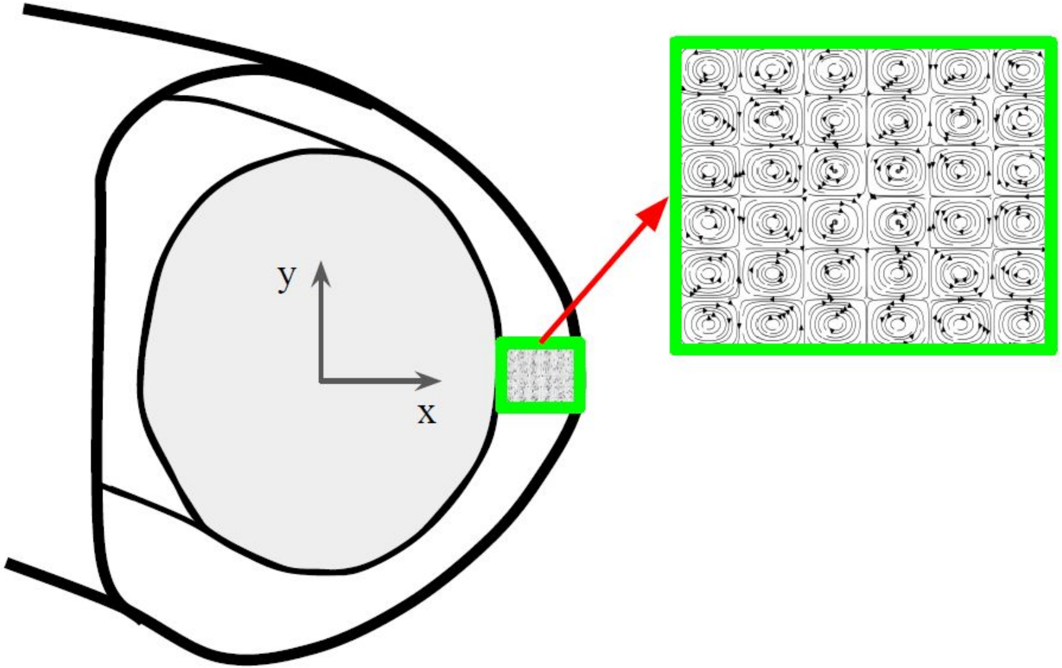
- ExB shear feedback, predator-prey
  - Zonal flows (predator) and turbulence intensity (prey)
- Jams (time-delay between temperature modulations and local heat flux)

But… is there an even **simpler** physical mechanism that can produce **layering**?

Answer: Yes (e.g., pattern of cells)

Some Questions:

- How does staircase beat homogenization?
- Is the staircase a meta-stable state?
- What is the minimal set of scales to recover layering?



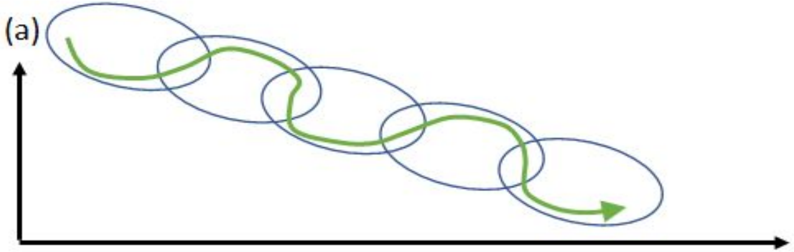
Fixed Cellular Array Problem

(another way to get a Staircase)

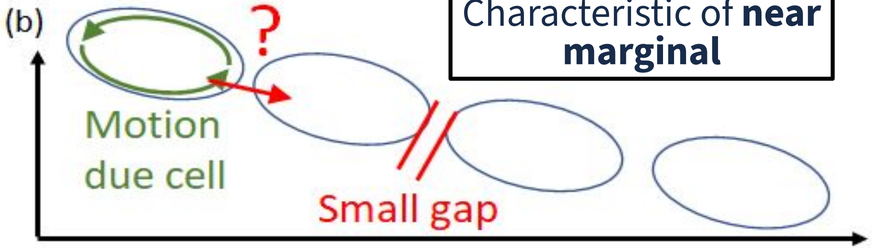
Marginally Overlapping Cells

Transport of particles between marginally overlapping cells (characteristic of near marginal) is an important topic in fusion plasma. Two different transport mechanisms:

<u>**Overlapping**</u>: particles can transport directly from cell to cell, wandering along streamlines



<u>**Nearly-overlapping**</u> (cells sit at near overlap): transport is a synergy of motion due to cells and random kicks (Collisional diffusion, ambient scattering) thru gap regions.



Characteristic of **near marginal**

Coexistence of:

~ **Fast** transport - Mixing in cell

~ **Slow** transport - kicks between cells (ambient diffusion)

Transport and Profile in FCA

Profile?

Consider concentration of injected dye (passive scalar transport in eddys) → profile

- Layering! Simple consequence of two rates
- $Pe = \tau_H / \tau_D >> 1$ (Necessary criteria, global transport hybrid)
- Staircase arises in stationary array of passive eddies (Note that there is no FEEDBACK)

Transport?

Answer: $Deff \sim [D\, D_{cell}]^{1/2}$ (Not a simple addition of process!)

Back-of-Envelope Calculation

$D^* \approx f_{active}((\Delta x)^2 / \Delta t);$

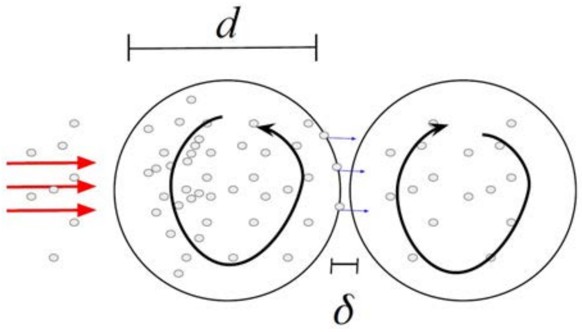
$f_{active} \equiv$ active fraction $\sim \delta / d$

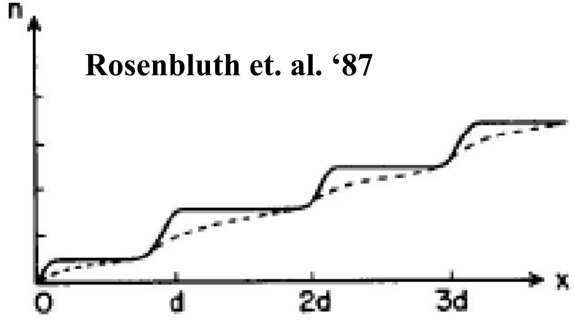
$\Delta t \sim d / U_o \rightarrow$ cell circulation time

So, $\delta^2 \sim D\, \Delta t \sim D\, d / U_o$

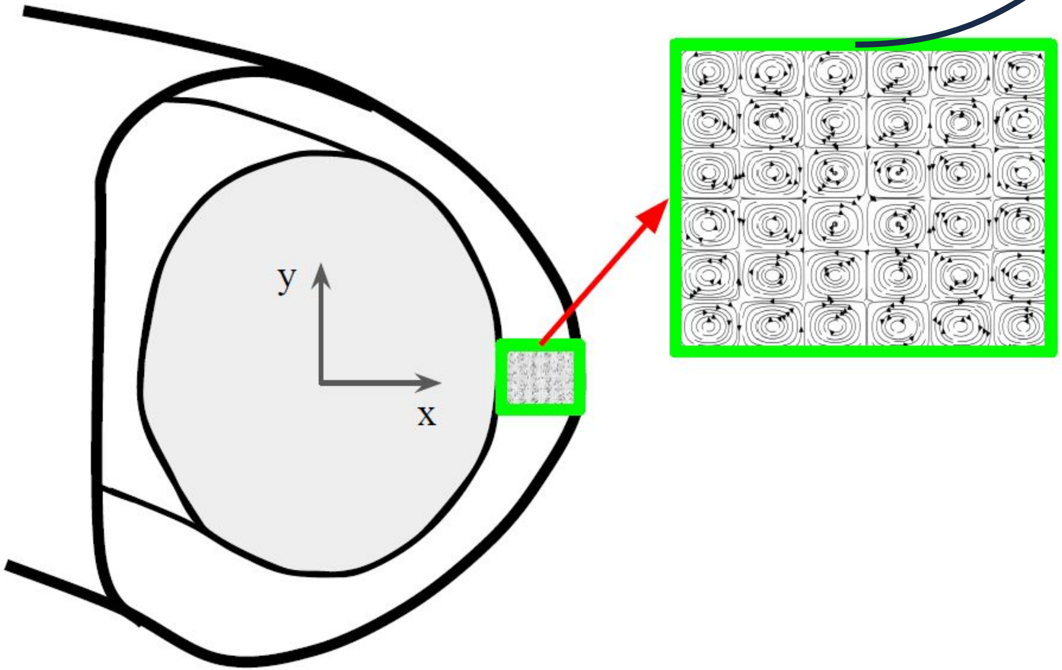
$D^* \sim [(D\, d / U_o)^{1/2}\, 1 / d]\, (d^2 / d)\, U_o \sim [D\, D_{cell}]^{1/2}$

Key: Closed cell boundaries are essential for layering to form.





"Steep transitions in the density exist between each cell."



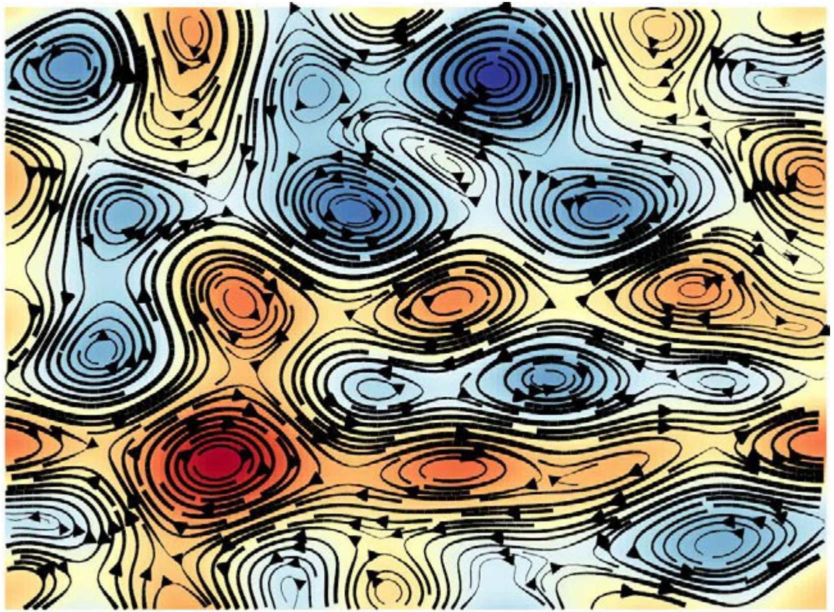
BUT, this setup is contrived, NOT self-organized!!! Cellular array is severely constrained!

Relaxing FCA w/ FVA

(What about the dynamics of a less constrained cell array?)

Consider a Broader Approach

Example of **less constrained** cell array



To answer these questions, we use the idea of a **Melting Vortex Crystal**…

- We want to study a much more **general** and **less constrained** version of the cell array (i.e., vortex array with fluctuations)
- How **resilient** is the staircase in the presence of these small variations to a fixed vortex array?

In the process of studying the resilience of the staircase, we aim to answer the following:

1. What happens to interspersed regions of strong scalar concentration mixing as cells relax? What about general cell interactions/behavior?
2. What is the behavior of the scalar trajectory through the vortex array?
3. How does the increase of scattering in the vortex array affect the transport of scalar concentration?

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For Passive Scalar Results in FVA see:

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Staircase resiliency in a fluctuating cellular array

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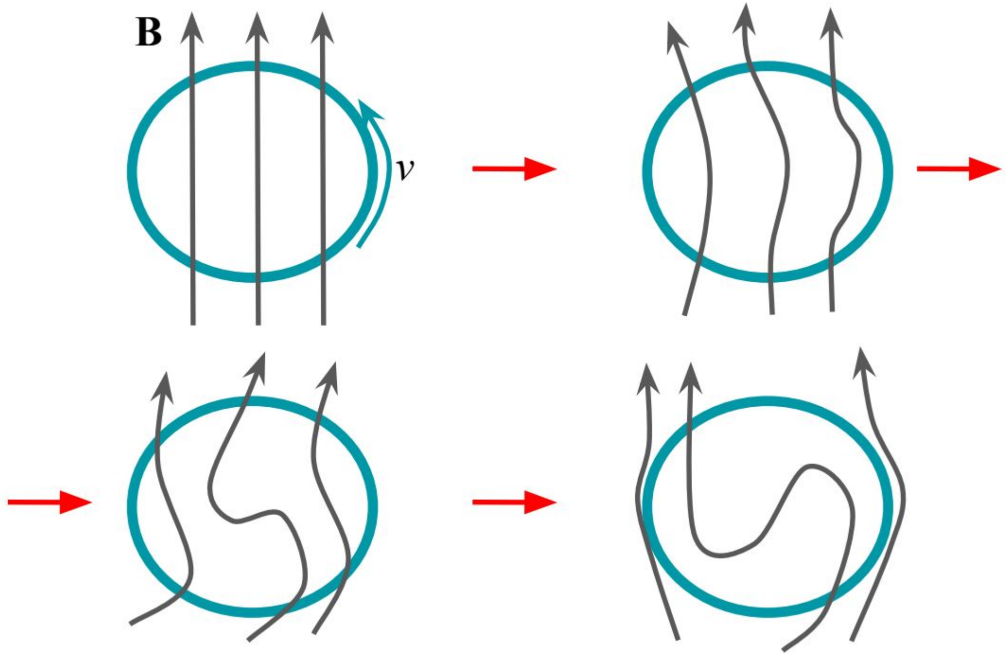

Active Scalar Dynamics

2D MHD system including feedback

Accepted manuscript (April 2025)

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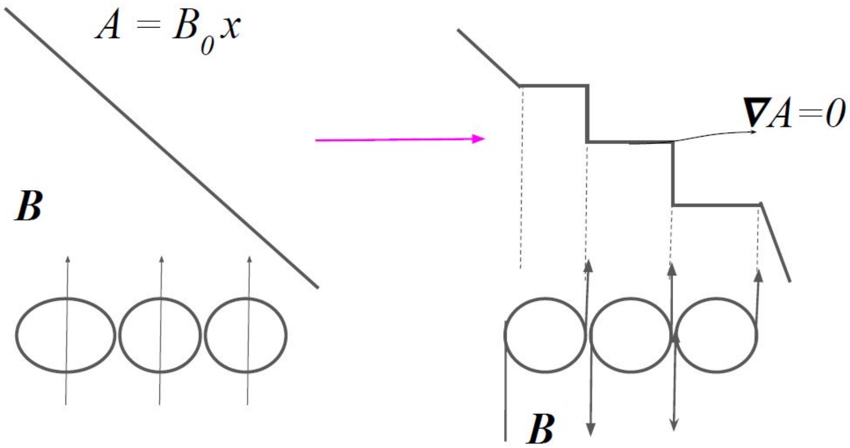
Active Scalar Dynamics

A logical next step to explore is the effects that an active scalar has on the cellular array and inhomogeneous mixing.

- Converting passive to active will result in effects such as flux expulsion
  - Flux expulsion is simplest dynamic problem in non-ideal MHD.



Why this model?

- ***B*** expelled to boundaries, thus holds cells together! → Rigid staircase.

Note: Strength of B_o plays an important role!

We turn passive scalar into an active scalar, creating a feedback between magnetic field and vortices:

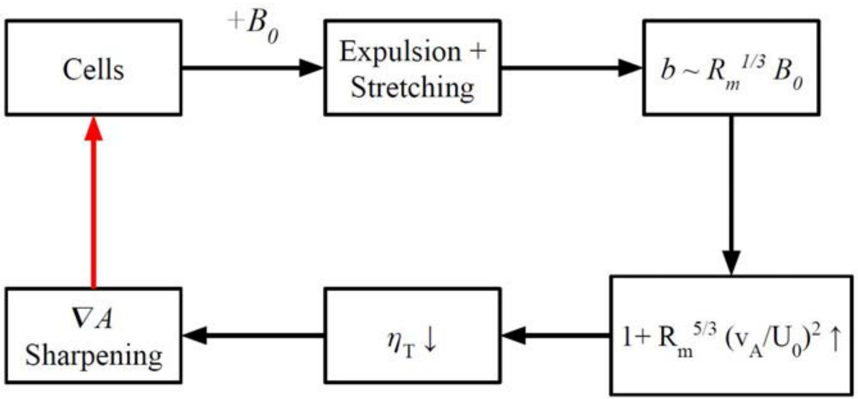
$$\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n - D\nabla^2 n = 0 \longrightarrow \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right) A = \frac{1}{R_m}\nabla^2 A \text{ -}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right) \omega = \frac{1}{\Omega}\nabla^2 \omega + M^2 \boxed{\left(\mathbf{B} \cdot \nabla \nabla^2 A\right)} + F_\omega$$

$$\boxed{\Omega = \frac{\tau_\nu}{\tau_H}} \qquad \boxed{M = \frac{v_A}{U_0}}$$

$$F_\omega \equiv -n^3 \left[\cos(nx) + \cos(ny)\right] / \Omega$$

B strengthens ∇A



But excessive values of $\boldsymbol{B}_0$ can disrupt vortex structures!

- What occurs then to layered structures?? Later..

During expulsion process, we enter a magnetic suppressive stage:

- $\boldsymbol{B}_0$ restrains turbulent transport

Transport across eddies quantified by η_T

- η_T decreases in suppressive stage

$$\eta_T \approx \frac{\eta_k}{1 + R_m \langle \mathbf{B} \rangle^2 / \langle u^2 \rangle},$$
$$\eta_k = \sqrt{\langle u^2 \rangle} L.$$

Cattaneo & Vainshtein '91

Note: suppression occurs for limited time (***B*** eventually decays in 2D).

So let's study what occurs to staircase structure for various values of $\boldsymbol{B}_0$ in a cellular array w/ intrinsic noise…

Magnetic Staircase ($\Sigma < 1$)

The two MHD scenarios are distinguished by $\Sigma < 1$ (flux expulsion) and $\Sigma \geq 1$ (cell disruption).

- **Σ is derived from the relative strength of the fluid convective term and the magnetic tension term.**

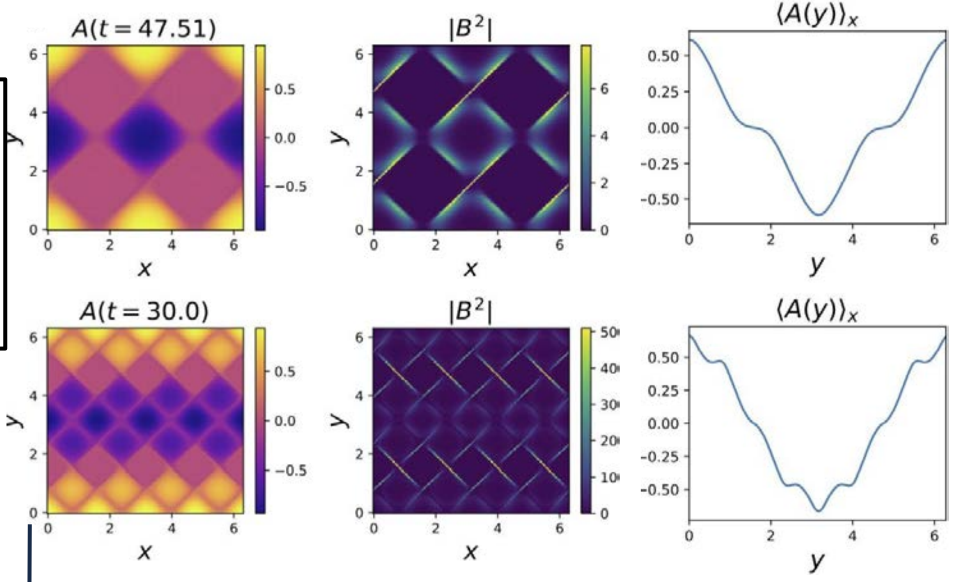
$$\Sigma = M^2 R_m$$

We study the process of inhomogeneous mixing by first initializing the active scalar as

$$A(x, y, t = 0) = A_0 \cos x$$

For all simulations, we fix R_m and only vary the values of B_0 and Ω.

- In $\Sigma < 1$, A homogenizes within regions of vortices, thus producing steps in the profile.
- Magnetic field lines are expelled at boundaries and hold cell structure together.



BUT what about $\Sigma > 1$ situations?

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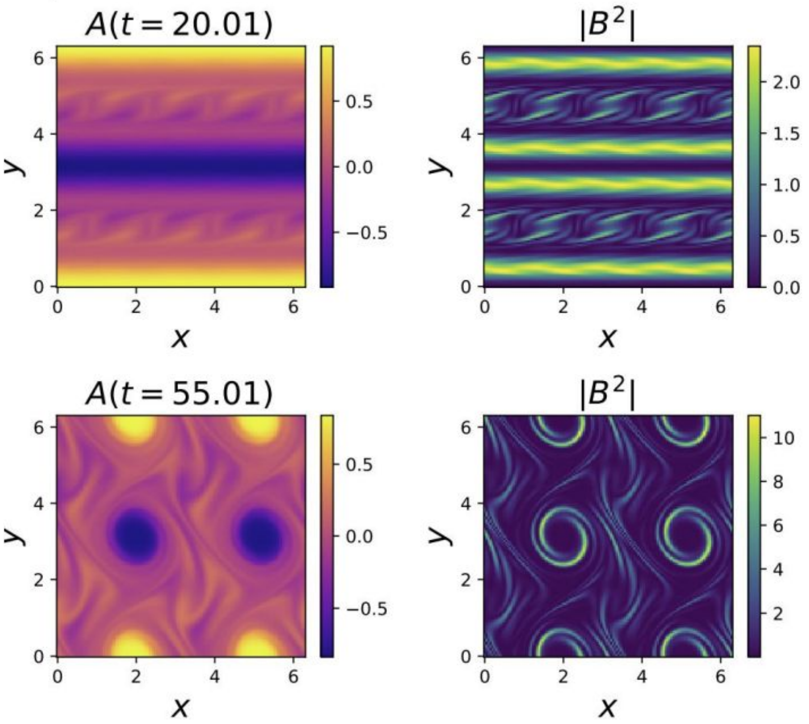
Magnetic Staircase ($\Sigma > 1$)

Contrary to prior believe, layered structure survives!

Prior Findings vs. New Results

- **Single-cell studies:**
  For $\Sigma > 1$, magnetic stresses disrupt and collapse the cellular structure.
- **Multi-cell system (this study):**
  The original vortex array collapses, *yet layering persists*.
- **Why does layering survive?**
  → **Answer:** Residual cells remain and continue to support localized homogenization and gradient sharpening.



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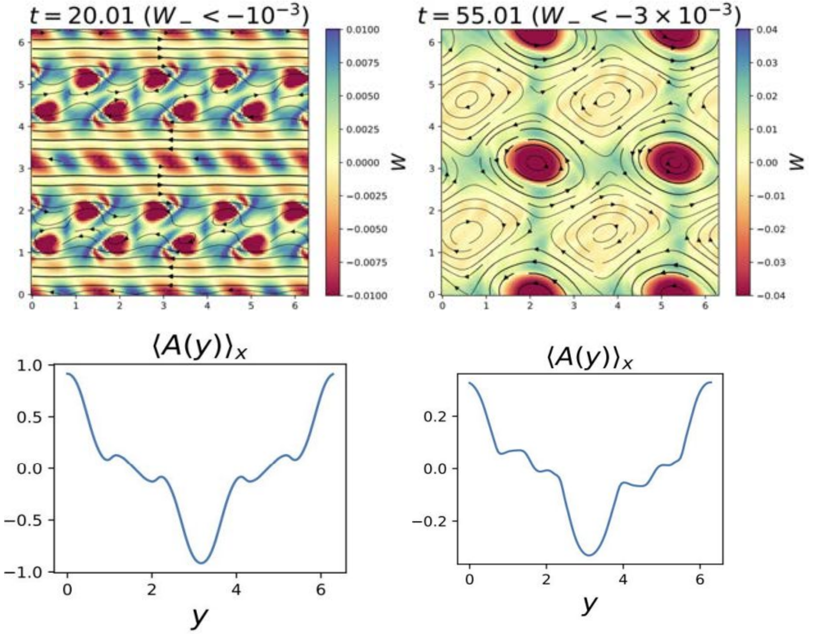
Residual Cells → Layering

Quantifying Residual Cell Structure

- We use the **Okubo-Weiss field** (W) to diagnose the local flow topology.
- W distinguishes **vortical regions** (rotation-dominated, $W < 0$) from **strain-dominated regions** ($W > 0$).

$$W = \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 - \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)^2$$
$$= W_+ - W_-.$$

- Even in the $\Sigma > 1$ regime where large-scale vortex structure collapses,
  → **Negative W regions reveal residual cell-like structures**.
- These residual cells are sufficient to sustain layering.



Key: **Layering still forms and persists in both flux expulsion and vortex disruption regimes!**

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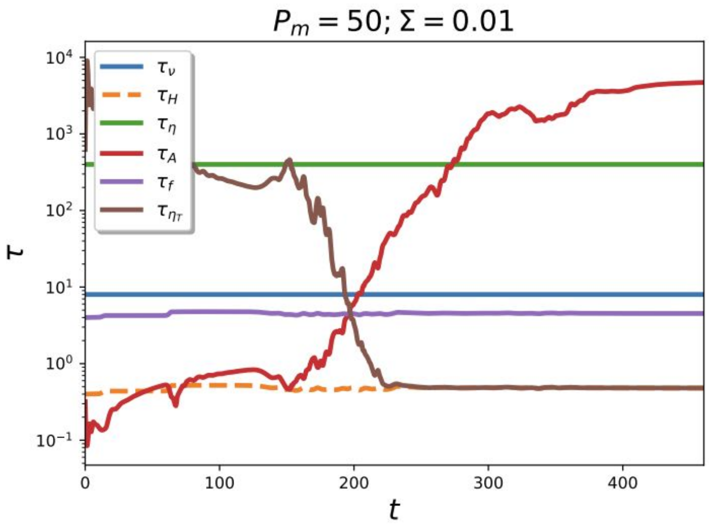
Comparison to Passive Scalar

Magnetic Case: Timescale Comparison via Eddy Resistivity

- In the active scalar (magnetic) case, **eddy resistivity** acts as the diffusive analog to molecular diffusivity in the passive case.

- We compare the corresponding **turbulent resistivity timescale** ($\tau_{\eta\mathrm{T}}$) to the **cell circulation timescale** (τ_{H})
 → as a **quantitative measure of staircase rigidity**.

- Result: The ratio $\tau_{\eta\mathrm{T}}/\tau_{\mathrm{H}}$ **increases by up to two orders of magnitude**,
 → indicating significantly enhanced rigidity compared to the passive case.



Scalar Type	Passive	Active
κ	$\mathcal{O}(10^0)$	$\mathcal{O}(10^{-3})$
$\tau_{\mathrm{slow}}/\tau_{\mathrm{fast}}$ timescale	$\tau_D/\tau_H \propto \mathcal{O}(10^1)$	$\tau_{\eta_T}/\tau_H \propto \mathcal{O}(10^3)$

However, **structure lifetime is limited in 2D MHD…**
 → due to the inevitable **resistive decay of the magnetic field**.

Okay, but problem of staircase lifetime remains…
Can we extend the life of the staircase?

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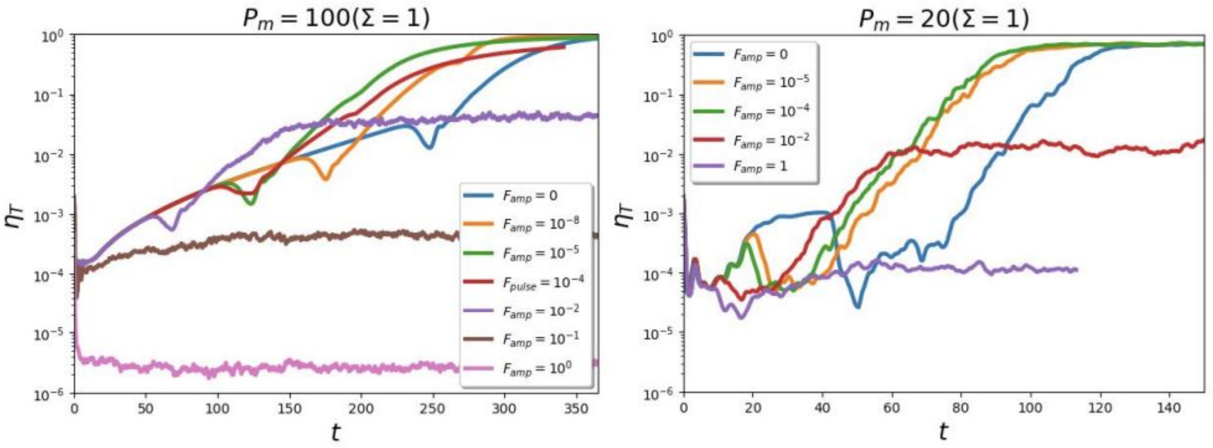
Stochastic Forcing

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\boldsymbol{\nabla}\right) A = \frac{1}{R_m}\nabla^2 A + \boxed{F_A}$$

$$\eta_T \sim \frac{\left(\langle \tilde{u^2} \rangle - \langle \tilde{F_a}^2 \rangle \tau_a\right)\tau}{1 + R_m \frac{\langle B^2 \rangle}{\langle u^2 \rangle}}$$



We conduct simulation runs with stochastic forcing, scanning across various forcing amplitudes.

- The key finding is that stochastic forcing prolongs transport suppression.
  - As the forcing amplitude increases, suppression is enhanced—confirmed by reductions in turbulent resistivity.

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Active Scalar Key Findings

1. Staircases persist in both the flux expulsion and vortex disruption limits. Surprisingly, in the latter case, residual vortex cells homogenize A.

2. Weak magnetic fields suppress turbulent diffusion of A, increasing the disparity between cell circulation and inter-cell transport times by a factor of $\sim 10^2$. This Rm-dependent feedback reinforces the staircase, distinguishing the active scalar case from its passive counterpart.

3. External stochastic forcing extends the lifetime of magnetic staircase structures.

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Thank you!