UC San Diego

How ‘the tail wags the dog’: physics of edge-core coupling by inward turbulence spreading

Mingyun Cao, P.H. Diamond

Department of Physics, University of California, San Diego

US Transport Task Force Workshop, April 25, 2025

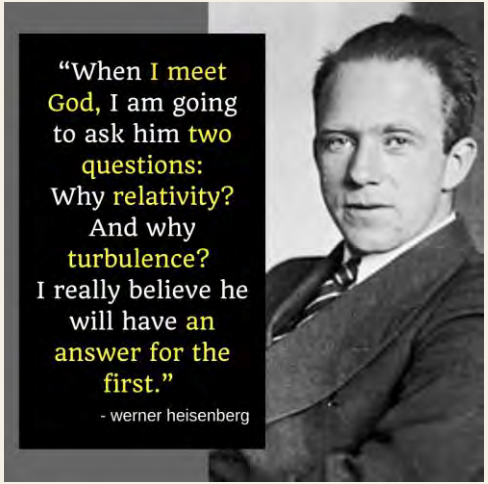
The work is supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences under Award Number DE-FG02-04ER54738.

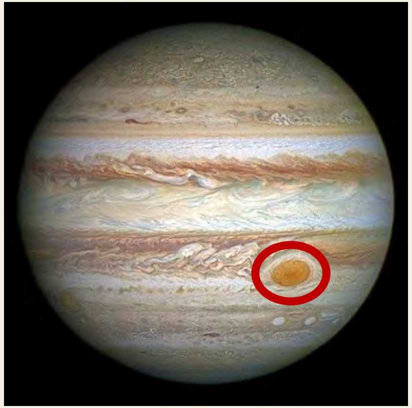
Contents

UC San Diego

Motivation 1: the missing piece of turbulence dynamics

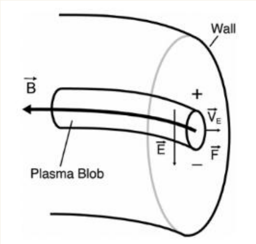
- Theorists: turbulence is the superposition of an ensemble of waves
- Turbulence is a multi-ingredient concoction—a ‘soup’: eddys, waves, structures, etc.
    - Structures: vortices, density blobs/voids, phase space holes
- The presence of structures is universal



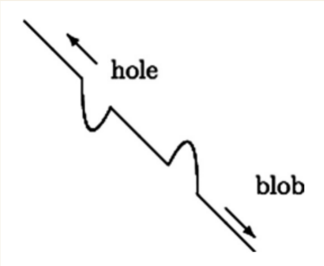


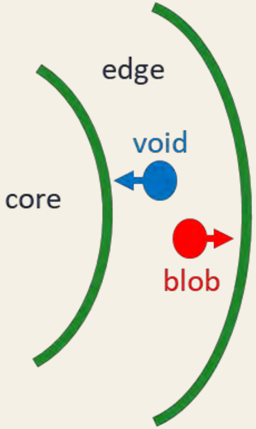
Motivation 1: the missing piece of turbulence dynamics

- **Coherent structures** are also present in plasma turbulence (tokamaks)
  - Density blobs/voids: plasma filaments with large +/- density fluctuations and long lifetime



- Existing studies (e.g.: scaling of blob convection velocity[1]) don't put coherent structures on equal footing as waves and zonal flows
- Another fact: there are **millions of papers on blobs**, but **very little attention on voids** ⇐ Difficulty in void diagnostics
- According to avalanche theory:
  - Particle conservation[2] ⇒ blobs/voids emerge in pairs
  - Joint reflection symmetry[3] ⇒ blobs and voids propagate in opposite direction, down and up the mean gradient





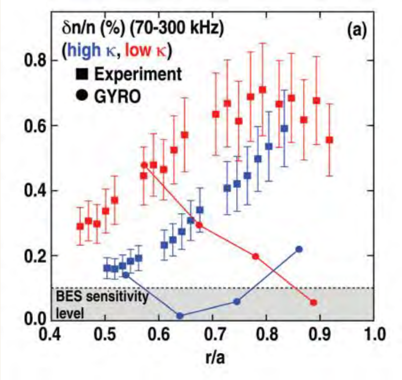
- It doesn't mean voids are less important.
  - Voids stay in the main plasma ⟶ **a messenger from edge to core**

1. S.I. Krasheninnikov et al., 2008. **2**. J.R. Myra et al., 2018. **3**. P.H. Diamond, T.S. Hahm, 1995.

UC San Diego

Motivation 2: physics of edge-core coupling

- To this end ⇒ a critical problem for optimal plasma performance: physics of edge-core coupling
- Feature of the edge-core coupling region: the fluctuation level predicted by **local** gyrokinetic simulation is lower than experimental observation—shortfall problem[1].
- A "known unknown": physics of what sets the width of the edge-core coupling region (no man's land, NML) ⇒ maybe the "excessive" turbulence comes from the edge?
- The story of "the tail (edge) wagging the dog (core)" has a long history:



"… And, finally, we have a very strong activity at the plasma edge. It controls the transition from one mode of confinement to another and its influence extends well into the bulk plasma… " —B.B. Kadomtsev, 1992

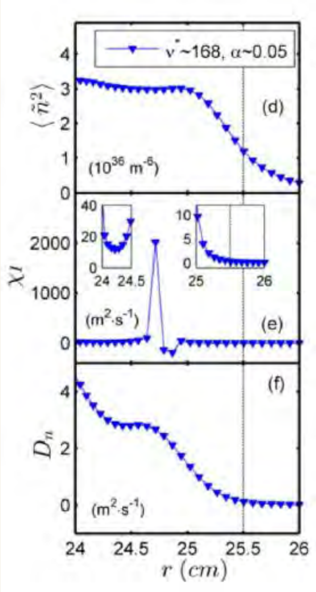
- **But no concrete picture or calculations other than vague discussions**
- **Can density void play a role in this process and address shortfall? Need more evidence**

1. C. Holland, et al., 2011
2. G. Dif-Pradalier, et al., 2022

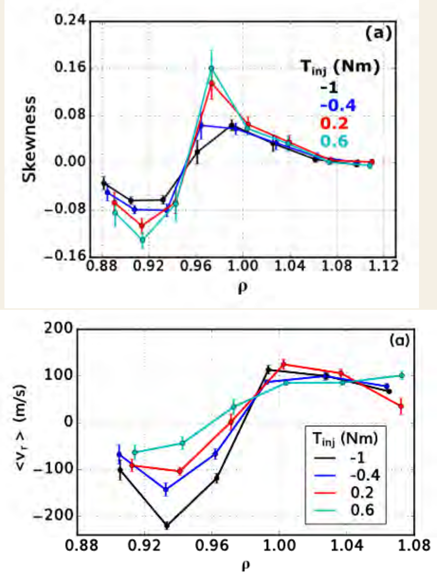
UC San Diego

Motivation 2: physics of edge-core coupling

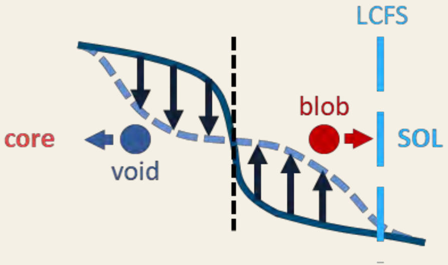
- Probes are applicable on devices operating in lower temperature[1].
- Beam emission spectroscopy (BES) provides us with more information on voids[2].



Long et al. on J-TEXT[1]



Filipp et al. on DIII-D[2]



We confirm:

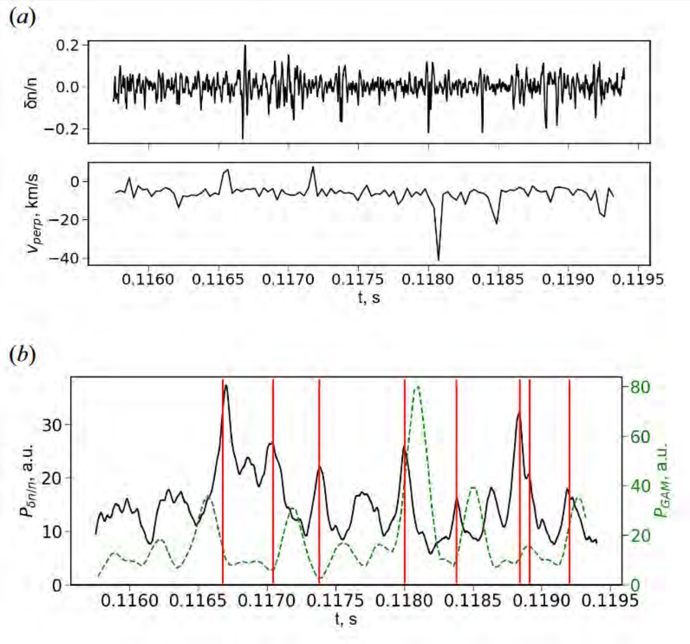
- Turbulence spreading in edge plasma is non-diffusive ⇒ the presence of coherent structures.
- Outward-moving blobs and inward-moving voids are created **in pairs** from edge gradient relaxation events (GREs) **close to LCFS**.
- **Voids stay in the main plasma** ⇒ voids are able to energize the NML.

1. Ting Long et al., 2024, NF.
2. Filipp Khabanov et al., 2024, NF.

UC San Diego

Motivation 2: physics of edge-core coupling

- More experimental results on how an inward moving void energizes the edge



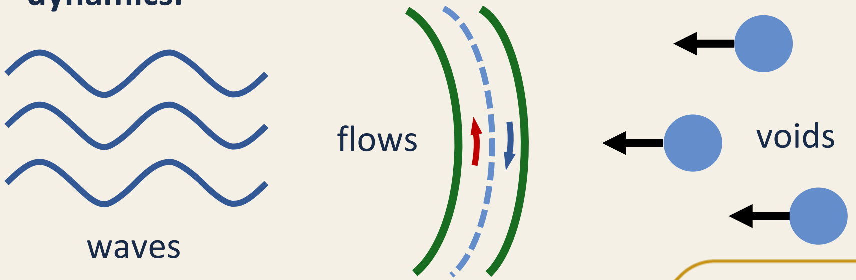
Alsu et al. on MAST[1]

- Bursts of zonal flow power usually follow the detection of density voids.

  ⇒ density voids can drive zonal flow.

- Message: inward moving density voids are important components of edge turbulence, which can interact with waves and zonal flows.

  ⇒ **Need a model to figure out the role hole plays in edge dynamics.**

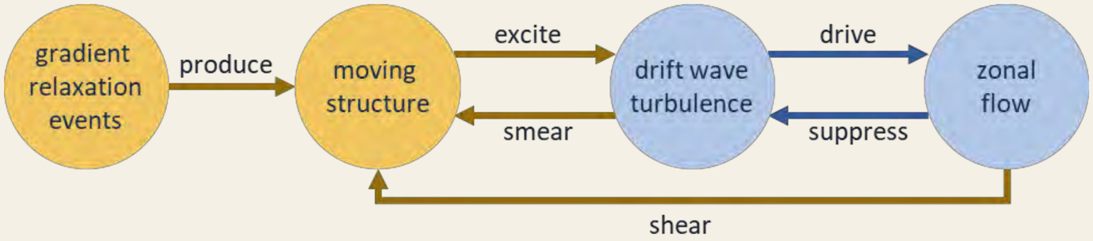


1. Alsu Sladkomedova et al., 2024, JPP.

UC San Diego

Scope & Preview

- Questions we aim to address:
  1. What is the width of the turbulent layer (no man's land) driven by the voids?
  2. What are the mechanism and shearing rate of the void-driven zonal flow?
  3. How do (ambient) turbulence and zonal flow affect density voids?



- Takeaways:
  - A moving void can excite drift wave turbulence and hence drive zonal flow.
  - The width of no man's land is of order $100\ \rho_s$ for typical parameters.
  - The shearing rate of the flow driven by the void could be $\gtrsim$ the ambient shearing rate.
  - Turbulence and shear flow can constrain the void lifetime, which is predicted to range from a few to 100 μs.

Model: Emission of drift waves from moving voids

- Develop a model from scratch: three incentives

Association

- An accelerating charge can emit EM wave → a moving void can be treated as a macro particle emitting waves.

Observation

- Radial propagation speed of voids is comparable to the diamagnetic drift velocity in the strong spreading scenario.[1]

Demand

- Connect to previous theoretical models of coherent structures, e.g., two-field model for structure convection.[2]

- Picture: the Cherenkov emission of drift waves from voids moving through the background plasma (recall dressed test particle model) ⇒ start from Hasegawa-Wakatani model.

1. T. Long et al., 2024, NF.
2. O.E. Garcia et al., 2005, PoP.

Model: Partition of the space

- Hasegawa-Wakatani model (with curvature drive):

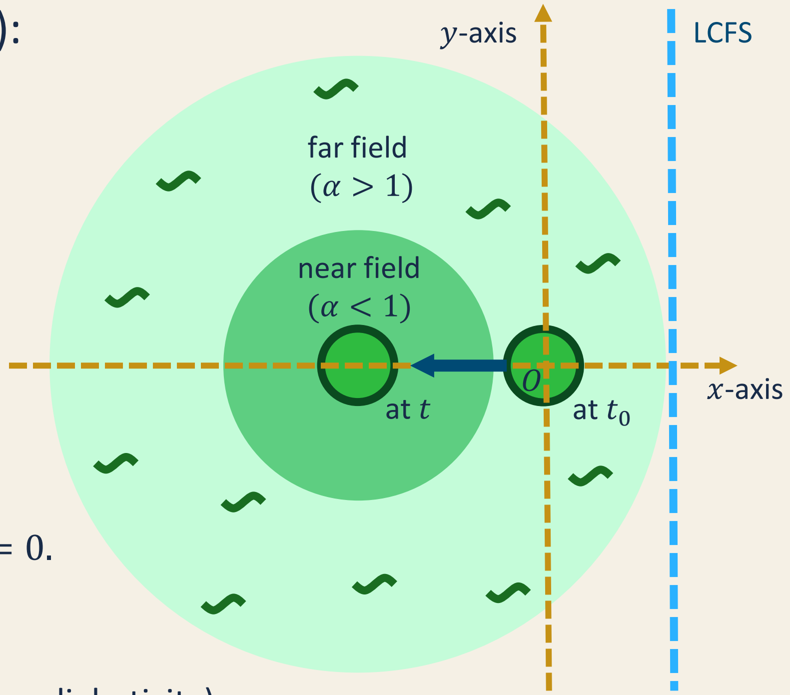
$$\frac{d}{dt}\nabla_\perp^2\varphi + \frac{2\rho_s}{R_c}\frac{1}{n_0}\frac{\partial n}{\partial y} = D_\parallel \nabla_\parallel^2\left(\frac{n}{n_0} - \varphi\right)$$

$$\frac{1}{n_0}\frac{dn}{dt} = D_\parallel \nabla_\parallel^2\left(\frac{n}{n_0} - \varphi\right)$$

- Divide the whole space into two parts:
  - Near field regime: close to the structure, $\alpha < 1$
    ($\alpha > 1 \rightarrow$ no density mixing $\rightarrow$ no structure formation)

    $\Rightarrow$ Two-field model[1]: $\frac{d}{dt}\nabla_\perp^2\varphi + \frac{2\rho_s}{R_c}\frac{1}{n_0}\frac{\partial n}{\partial y} = 0, \qquad \frac{1}{n_0}\frac{dn}{dt} = 0.$
  - Far field regime: far away from the structure, $\alpha > 1$

    $\Rightarrow$ Hasegawa-Mima equation: $\frac{d}{dt}\nabla_\perp^2\varphi - \frac{1}{n_0}\frac{dn}{dt} = 0$ (α: adiabaticity)



1. O.E. Garcia et al., 2005, PoP.

UC San Diego

Model: local solutions of the far field eqn

- Target: the turbulence field excited by a moving void $\Rightarrow$ focus on the far field regime ($\alpha > 1$)
- Void enters the model via profile modulation, i.e., $n = n_0 + n_v + \tilde{n}$ (Akin to test particle model)

$$\frac{d}{dt}(\nabla_\perp^2 \varphi - \varphi) - v_* \frac{\partial \varphi}{\partial y} = \boxed{\frac{1}{n_0}\frac{dn_v}{dt}} \longrightarrow \text{source} \qquad n_v = 2\pi n_0 h \Delta x \Delta y \delta(x + u_x t)\delta\left(y - u_y t\right) H(t) H(\tau_v - t)$$

h: magnitude; $\Delta x, \Delta y$: spatial extent; u_x, u_y: convection speed; τ_v: lifetime

- For tractability, we employ the delta-function shaped expression for the void.
- Workflow of the rest of the calculations:

Get the Green's func of the linearized H-M eqn and then solve φ of the far field equation → Estimate the void-induced turbulence intensity flux and width of the no man's land → Compare the shearing rate of the structure-driven flow to that of the ambient flow

UC San Diego

Model: local solutions of the far field eqn

- One trick we play in the derivation:

$$\frac{\partial}{\partial t} \longleftarrow \boxed{\left(\frac{\partial}{\partial t}+\underbrace{\boxed{\boldsymbol{v}\cdot\nabla}}_{\text{far field}}\right)}(\nabla_\perp^2\varphi-\varphi)-v_*\frac{\partial\varphi}{\partial y}=\frac{1}{n_0}\boxed{\left(\frac{\partial}{\partial t}+\underbrace{\boxed{\boldsymbol{v}\cdot\nabla}}_{\text{near field}}\right)n_v} \longrightarrow -\frac{n_v}{\tau_v}$$

Recall n_v is spatially-localized.

- We linearize the L.H.S. while retain the total derivative and approximate it as $-n_v/\tau_v$.

  The reason we can adopt a "double standard": the convection term on the L.H.S. corresponds to the far field, while the convection term on the R.H.S. corresponds to the near field due to the spatial localization of the void.

- The linearization of the Hasegawa-Mima equation (i.e., bare propagator) is strictly valid in the $Ku < 1$ regime. For $Ku \gtrsim 1$, waves undergo strong scattering $\Rightarrow$ need to consider renormalized propagator.

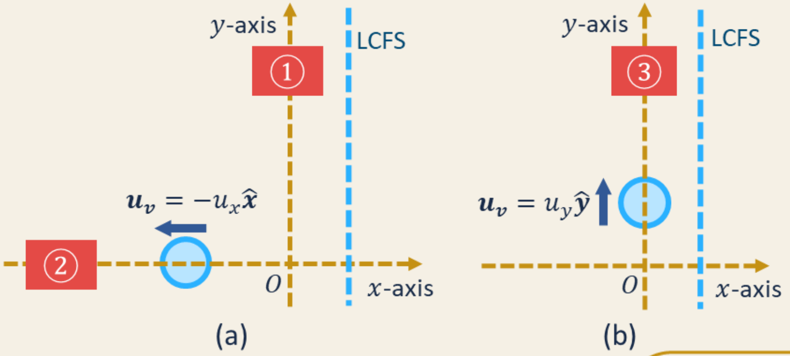
Model: solutions of three limiting cases

- We get the desired Green's function from geophysics, as Rossby wave equation with a finite Rossby deformation radius is homotopic to the H-M eqn. (Surprisingly little literature on Green's function of H-M eqn!) Still meet two challenges:
  - The Green's function is complicated:

$$G = -\int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i} \exp\left(s\tau + \frac{v_* \chi}{2s}\right) \frac{1}{2\pi s} \mathrm{K}_0\left[\left(1 + \left(\frac{v_*}{2s}\right)^2\right)^{1/2} \rho\right].$$

$$\tau = t - t'$$
$$\chi = y - y'$$
$$\rho = |\boldsymbol{r} - \boldsymbol{r}'|$$

  - Voids move in both poloidal and radial directions.
- Solution: consider three limiting cases:
  a) Radially moving void $(u_y = 0)$:
     1) away from the x-axis ($|y| \gg |x|$)
     2) near x-axis ($|x| \gg |y|$)
  b) Poloidally moving void $(u_x = 0)$:
     3) near y-axis ($|y| \gg |x|$)



UC San Diego

Model: solutions of three limiting cases

- In the limit of $\tau \to \infty$, the asymptotic form of the Green's function is

$$G \to -\frac{1}{2\pi}\frac{1}{\sqrt{v_*\rho\tau}}\cos\left[\sqrt{2v_*(\rho-\chi)\tau}\right].$$

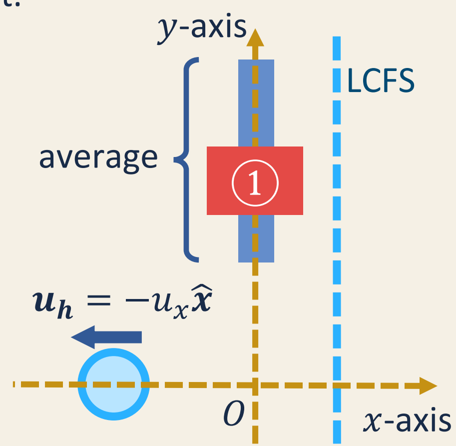
  - For causality: the influence of the void should be confined to $\rho \lesssim v_*\tau$.
  - $u_x, u_y \lesssim v_*$ so that the perturbation excited by void could propagate ahead of it.
- Case 1: radially moving void, away from x-axis
  - Spatial-temporal ordering:

$$x \lesssim x' \sim d_{pe}(= u_x\tau_v) \sim \Delta x \sim \Delta y \ll y,$$
$$1/\omega_{ci} \ll 1/\omega_* \ll t' \sim \tau_v \ll t.$$

  - Electrostatic potential φ:

$$\varphi = \frac{2h\Delta x\Delta y}{v_*u_x\tau_v t}\sin\left[\left(\frac{v_*t}{y}\right)^{\frac{1}{2}}\frac{d_{pe}}{2}\right]\cos\left[\left(\frac{v_*t}{y}\right)^{\frac{1}{2}}\left(x+\frac{d_{pe}}{2}\right)\right]$$

  - $\tilde{\boldsymbol{v}} = -\nabla\varphi \times \hat{\boldsymbol{z}} \Rightarrow \omega_s^v = -\int\langle\tilde{v}_x\tilde{v}_y\rangle'' dt$ ⟨ ⟩: local poloidal average



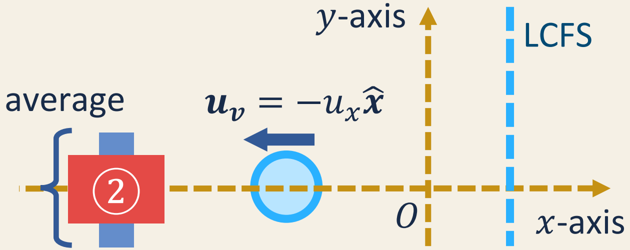
Model: solutions of three limiting cases

- Case 2: radially moving void, near x-axis
  - Spatial temporal ordering:

$$y \lesssim x' \sim d_{pe} \sim \Delta x \sim \Delta y \ll x$$
$$1/\omega_* \ll t' \sim \tau_v \ll t.$$

  - Electrostatic potential φ:

$$\varphi \approx -\frac{\sqrt{2}h\Delta x\Delta y}{v_* u_x \tau_v t} 2\cos\left([-2v_* t(x+y)]^{1/2} + \left[\frac{-v_* t}{2(x+y)}\right]^{1/2}\frac{u_x\tau_v}{2}\right)\sin\left(\left[\frac{-v_* t}{2(x+y)}\right]^{1/2}\frac{u_x\tau_v}{2}\right)$$

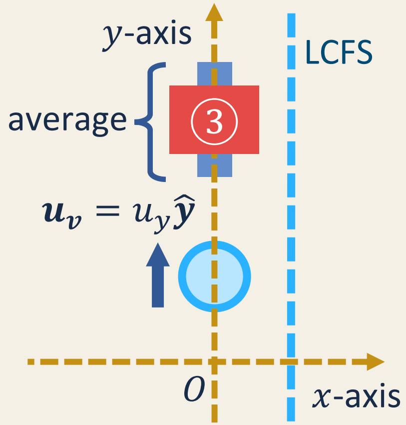


- Case 3: poloidally moving void, near y-axis
  - spatial-temporal ordering:

$$x \lesssim y' \sim u_y\tau_v \sim \Delta x \sim \Delta y \ll y, 1/\omega_* \ll t' \sim \tau_h \ll t$$

  - Electrostatic potential φ

$$\varphi \approx \frac{\pi h\Delta x\Delta y}{2k_0 u_y \tau_v} \mathrm{J}_0\left\{\left[\left(k_0 y - \frac{k_0 v_* t}{1+k_0^2}\right)^2 + k_0^2 x^2\right]^{1/2}\right\}$$



UC San Diego

Results: void-induced turbulence intensity flux

- After generated at ψ_0, voids move inward and stop at ψ_1. But the turbulence they excite could propagate deeper into the main plasma ($\sim \psi_2$)
- (ψ_2, ψ_1) defines the core-edge coupling region (no man's land)
- The balance equation for turbulence intensity (without dissipation):

$$\frac{\partial}{\partial t}\langle \tilde{v}^2 \rangle = -\frac{\partial}{\partial x}\langle \bar{\Gamma} \rangle + \kappa \langle \tilde{v}\tilde{n} \rangle \quad (\kappa\text{: curvature})$$

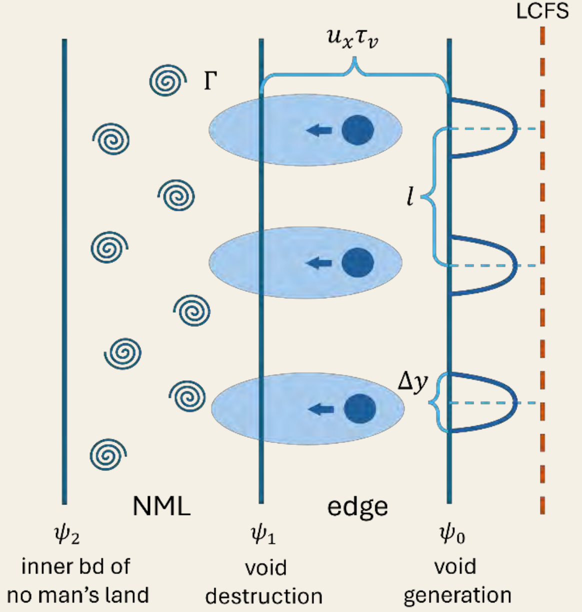
- $\langle \bar{\Gamma} \rangle$ is the turbulence intensity flux after zonal and time average.
- Integrating over the NML, the ratio of the turbulence intensity flux induced by voids to the total local production in no man's land is

$$R_a = \frac{\langle \bar{\Gamma} \rangle|_{\psi_1}}{\int_{\psi_2}^{\psi_1} \kappa \langle \tilde{v}\tilde{n} \rangle dr} \approx \frac{\langle \bar{\Gamma} \rangle|_{\psi_1}}{\kappa \langle \tilde{v}\tilde{n} \rangle w_{nml}}$$

- $\langle \bar{\Gamma} \rangle|_{\psi_2}$ is neglected as the core remains unaffected by voids.
- In NML, $Ra \sim 1 \Rightarrow$ defines the NML width: $w_{nml} \sim \langle \bar{\Gamma} \rangle|_{\psi_1} / \kappa \langle \tilde{v}\tilde{n} \rangle$

l: spacing between emitters

Δy: width of emitters.



UC San Diego

Results: the width of the NML

- Edge instabilities (GREs) contain N troughs (N void emitters).
- After each waiting time τ_w, N voids are simultaneously generated. Each void provides a turbulence intensity burst $\Delta I \Rightarrow \Gamma$ is the superposition of these pulses

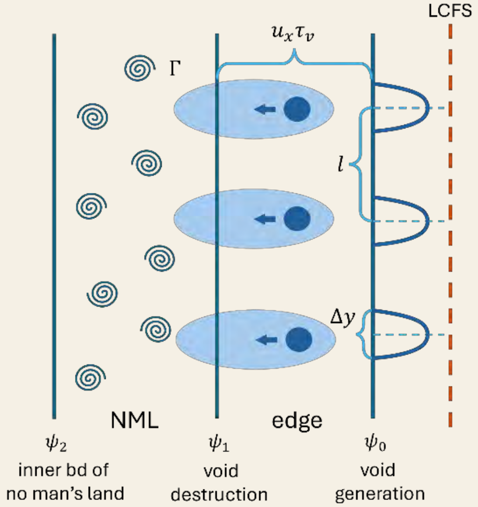
$$\Gamma = \sum_{i,j} u_x \Delta I \exp\left[-\frac{(y-il)^2}{2\Delta y^2}\right] \exp\left[-\frac{(t-j\tau_w)^2}{2\tau_v^2}\right] \sim \sum_{i,j} u_x \Delta I 2\pi \Delta y \tau_v \delta(y-il)\delta(t-j\tau_w)\,.$$

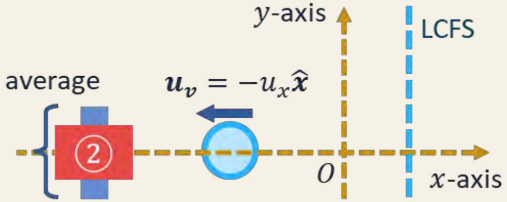
- ΔI could be evaluated from the local solution at $x \to \rho_1^-$ in case 2. After spatial and temporal averaging:

$$\langle \bar{\Gamma} \rangle\Big|_{\rho_1} \approx 2\pi \left(\frac{h\Delta x \Delta y}{u_x \tau_v}\right)^2 \frac{1}{v_* \tau_v^2} \frac{N\Delta y}{L_y} \frac{\tau_v}{\tau_w},$$

$$\longrightarrow \quad w_{nml} \sim \frac{2\pi}{\kappa \langle \tilde{v}\tilde{n} \rangle} \left(\frac{h\Delta x \Delta y}{u_x \tau_v}\right)^2 \frac{1}{v_* \tau_v^2} \frac{N\Delta y}{L_y} \frac{\tau_v}{\tau_w}.$$

- w depends on h, Δx, Δ_y, τ_w, which can be physically mapped to the amplitude, spatial scale, and frequency of GREs.
- For $N \sim \mathcal{O}(1)$ (strong ballooning), $\Delta x \sim \Delta y \sim 10$, $u_x \sim v_* \sim 10^{-2}$, $\tau_v \sim 10^3$, $l \sim 10^3$, $\tilde{v} \sim \tilde{n} \sim 10^{-2}$, $\kappa/2\pi \sim 10^{-4}$, $h \sim .1 \to$ $\boldsymbol{w_{nml} \sim 10^2 \rho_s}$.





UC San Diego

Results: shearing rate of void-driven flow

- Summary of the spatial-temporal orderings and shearing rates of the flow in these three cases :

Case	ω_s^v/ω_s^a	If $v_F^a \sim v_*$, $\Delta_F^a \sim 10\rho_s$
$\boldsymbol{v_h} = -u_x\hat{\boldsymbol{x}}$ away from x-axis	$\frac{\omega_s^h}{\omega_s^a} \sim \left(\frac{h\Delta x\Delta y}{v_* u_x \tau_v a}\right)^2 \frac{\Delta_F^a}{v_F^a/v_*}$	$\frac{\omega_s^v}{\omega_s^a} \sim 10h^2$
$\boldsymbol{v_h} = -u_x\hat{\boldsymbol{x}}$ near x-axis	$\frac{\omega_s^h}{\omega_s^a} \sim \left(\frac{h\Delta x\Delta y}{v_* u_x \tau_v}\right)^2 \frac{2\ln(a/v_*)\,\Delta_F^a}{x^3 v_F^a/v_*}$	$\frac{\omega_s^v}{\omega_s^a} \sim (10h)^2 \ \left(\frac{x}{\rho_s} \sim 10^2\right)$
$\boldsymbol{v_h} = u_y\hat{\boldsymbol{y}}$ near y-axis	$\frac{\omega_s^h}{\omega_s^a} \sim \frac{\pi(1+k_0^2)}{4k_0}\left(\frac{h\Delta x\Delta y}{v_* u_y \tau_v}\right)^2 \frac{x}{a^3}\frac{\Delta_F^a}{v_F^a/v_*}$	$\frac{\omega_s^v}{\omega_s^a} \sim h^2 \ \left(\frac{x}{\rho_s} \sim 10, k_0 = 1\right)$

- As $h = n_h/n_0 \in (0.1,1)$, in all cases, ω_s^vcould be comparable to ω_s^a (exceed it in case 2).
- It is order of magnitude estimate… But the flexibility in our choice of parameters indicate that sufficient large w and ω_s^v should exist in a considerably large portion of the parameter space.

x', y', t': integration coordinates; x, y, t: far-field coordinates; v_F^a: ambient flow velocity; Δ_F^a: ambient flow width
a: minor radius;ω_s^v: shearing rate of the structure-driven flow; ω_s^a: shearing rate of the ambient flow
In dimensionless form: $v_*/c_s \sim u_x/c_s \sim 2u_y/c_s \sim 10^{-2}$, $a/\rho_s \sim 10^3$, $\omega_{ci}\tau_v \sim 10^3$, $t \sim 10^5$

Results: estimate of the void lifetime

- To close the feedback loop: what are the effects of turbulence and flow on the structure?

  ⇒ Turbulence/flow can smear/shear the void, thus constraining its lifetime.

- Consider a diffusion model:

$$\partial_t n_v = D\nabla_\perp^2 n_v$$

- A practical definition of the void lifetime:

  ⇒ When h decays by half, void is vanished ⇒ $\tau_v = 2\Delta x^2/D$.

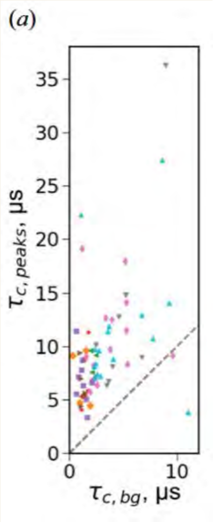
- For $\rho_* = \rho_s/L_n \sim .01,\ \omega_s^a/\omega_* \sim \rho_*^{1/2},\ \omega_*/\omega_{ci} \sim \rho_*$:
  - In purely diffusive regime ($\omega_s^a < Dk_\perp^2$ or $\frac{1}{2} < \delta < 1$):

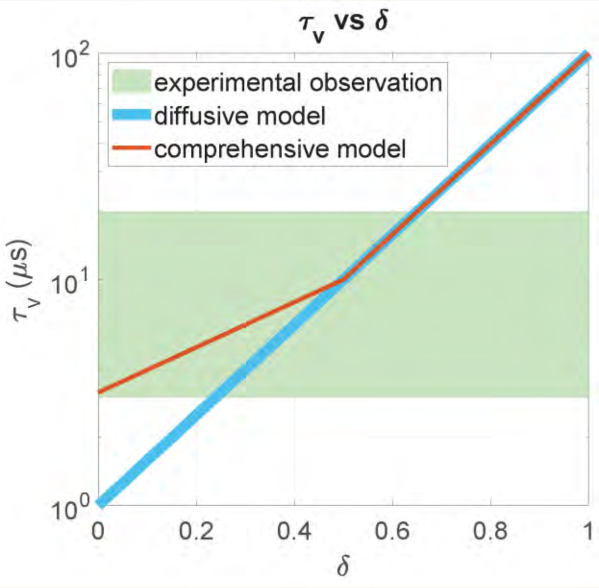
$$D/D_B = \rho_*^\delta,\ \tau_v \propto \rho_*^{-\delta}. \quad l_{mix} = L_n\rho_*^\delta$$

  - In shearing dominant regime ($\omega_s^a > Dk_\perp^2$ or $0 < \delta < \frac{1}{2}$):

$$D/D_B \sim \rho_*^{(1+2\delta)/4},\ \tau_v \propto \rho_*^{-(1+2\delta)/4}.$$

  - Our estimate: $\tau_v \sim 3 - 100\ \mu s$ vs. experiment: $\tau_v \sim 3 - 20\ \mu s$.





UC San Diego

Conclusion

- We develop a theory incorporating density voids into edge dynamics, which goes well beyond the traditional drift wave-zonal flow binary paradigm.
- We propose a realistic physical picture of how the tail (edge) wags the dog (core): the Cherenkov emission of drift waves from inward-moving voids drives substantial inward turbulence spreading, and so drives a broad turbulent layer.
- More specifically:
  - The width of the NML, which depends on the void parameters, is of order $100\ \rho_s$.
  - The shearing rate of the void-driven zonal flow is comparable to or even exceeds the ambient shear.
  - The void lifetime ranges from a few to $100\ \mu s$, which encompasses experimental values reasonably well.
- We expect that our model applies not only to L-mode, but also provides insights into H-mode, as edge-localized modes can be considered as a sort of GREs to some extent[1]

1. Nami Li, et al., 2023, NF.

UC San Diego

Future

- We suggest several possible directions for future research:
  - **For theories:**
    1. the net effect of voids on edge transport? decorrelation time vs shearing rate?
    2. a fully self-consistent model? Voids lose energy by radiation $\Rightarrow$ smaller τ_v
  - **For experiments:**
    1. correlation between the frequency of GREs and the turbulence level in no man’s land.
    2. direct evidence of void-turbulence/flow interactions using wavelet bispectrum analysis.
  - **For simulations:** To be aware of the importance of GREs, as inward moving void energizing the edge. Origination of shortfall in local gyrokinetic simulation: the absence of GREs, void generation, and void-induced turbulence spreading.

Thank you!