Transport Physics of Density Limits

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Outline

- Selected OV of density limit physics (L-mode)
 - Focus: role of particle transport
 - Emphasize: fluctuation studies $\leftarrow \rightarrow$ role of edge shear layer
- Theory of shear layer collapse
 - Shear flow production and its decline
 - Key: electron adiabaticity
- Desperately seeking Greenwald
 - What of current scaling?
 - Tokamak vs. RFP vs. Stellarator
- Thoughts for experiments

A Look at Density Limit Phenomenology

Density Limits: Some Basic Aspects

- Not a review!
- Greenwald density limit:



Constrains tokamak Operating Space

- Manifested on other devices
 - See especially <u>RFP</u> ($n \sim I_p$ scaling)

- Line averaged limit
- (Too) simple dependence!?
- Begs origin of *I_p* scaling?!
 Stellarators?
- Most fueling via edge → edge transport critical to n
 imits



- Argue: Edge Particle Transport is crucial
 - 'Disruptive' scenarios <u>secondary</u> outcome, largely consequence of <u>edge</u> <u>cooling</u>, following fueling vs. increased particle transport
 - \bar{n}_{g} reflects fundamental limit imposed by <u>particle transport</u>
- A Classic Experiment (Greenwald, et. al.)



- Density decays without disruption after shallow pellet injection
- \bar{n} asymptote scales with I_p
- Density limit enforced by transport-

induced relaxation

- Relaxation rate not studied

• More Evidence for Role of Edge Transport



- Post-pellet density decay time vs $\overline{J}/\overline{n}$.
- Increase in relaxation time near (usual)

limit: $\bar{J}/\bar{n} \sim 1+$

- Pellet in DIII-D beat \bar{n}_g
- Peaked profiles ← → enhanced core
 particle confinement (ITG turbulence
 reduced?)
- Reduced particle transport → impurity accumulation

(N.B. Deeper deposition)

Conventional Wisdom (Rogers + Drake '98, et seq.)

Reduced Fluid Simulation (no heat source)



- D+R on n-limit physics:
 - State of high $\nabla P, \beta$, cool electrons
 - DWT → resistive ballooning turbulence
 - Issue: Density limit vs beta limit??

$$\alpha_{MHD} = -Rq^2 d\beta/dr$$

 $\leftrightarrow \forall P \rightarrow ballooning drive$

$$\alpha_d = \rho_S C_s t_0 / L_n L_0$$

$$t_0 = \frac{(RL_n)^{\frac{1}{2}}}{c_S}$$

$$L_0 = 2\pi q \left(\frac{\nu_e R \rho_s}{2\Omega_e}\right)^{1/2}$$

→ Hybrid of drift frequency and collisionality

Density limit ←→ **Fluctuation Structure**



C-Mod profiles, Greenwald et al, 2002, PoP

- <u>Average</u> plasma density increases as a result of edge fueling → edge transport crucial to density limit.
- As *n* increases, high ⊥ transport region extends inward and fluctuation activity increases.
- Turbulence levels increase and perpendicular particle transport increases as $n/n_G \rightarrow 1$.

Recent Experiments - 1

(Y. Xu et al., NF, 2011)

<n_>=1.5x10¹⁹m⁻³ 0.8 0.6 0.4 <n_>=3.0x10 0.2 رک ک 0.0 -0.2-0.4-0.6-1000 100 200 -200 (a) lag (µs) 0.8 с×х 0.4 0.2 3.5 4.5 4 Line average density (10¹⁹ m⁻³) (b)

MAX

LRC vs \bar{n}

- Decrease in maximum correlation value of LRC (i.e. ZF strength) as line averaged density \bar{n} increases at the edge (r/a=0.95) in both **TEXTOR** and **TJ-II**.
- At high density ($\langle n_e \rangle > 2 \times 10^{19} \, m^{-3}$), the ٠ LRC (also associated with GAMs) drops rapidly with increasing density.
- The reduction in LRC due to increasing density is also accompanied by a reduction in edge mean radial electric field (Relation to ZFs).

Is density limit related to edge shear decay?

Recent Experiments - 2

(Schmid, Mans et al., PRL, 2017) – stellarator experiment

Eddy Tilt (a) С 1.00 $\alpha = -0.23 \pm 0.02$ $_{\rm ZF}/P_{\rm total}$ 0.10 0.01 0.1 1.0 10.0 \mathbf{C} P_{ZF}/P_{tot}

- Experimental verification of the importance of collisionality for large-scale structure formation in TJ-K.
- Analysis of the Reynolds stress shows a decrease in coupling between density and potential for increasing collisionality → hinders zonal flow drive (Bispectral study)
- Decrease of the zonal flow contribution to the total turbulent spectrum with collisionality *C*.
- a) Increase in decoupling between density (red) and potential (blue) coupling with collisionality C.
- b) Increase in ZF contribution to the spectrum in the adiabatic limit $(C \rightarrow 0)$
- $C \Leftrightarrow a diabaticity k_{\parallel}^2 V_{th}^2 / \omega v$

Recent Studies, Hong, et. al. (NF 2018)



- Joint pdf of $\tilde{V}_r, \tilde{V}_\theta$ for 3 densities, $\bar{n} \to n_g$
- $r r_{sep} = -1cm$
- Note:
 - Tilt lost, symmetry restored as $\overline{n} \rightarrow \overline{n}_g \rightarrow Weakened$ shear flow
 - Consistent with drop in P_{Re}

production by Reynolds stress

Key Parameter: Electron Adiabaticity



Electron adiabaticity $\alpha = \frac{k_{||}^2 v_{th}^2}{|\omega| v_{ei}}$ emerges as interesting local parameter. $\alpha \sim 3 \rightarrow 0.5$ during \bar{n} scan!

c)

 $\overline{2}$

Particle flux \uparrow and Reynolds power $P_{Re} =$ $-\langle V_{\theta} \rangle \partial_r \langle \tilde{V}_r \tilde{V}_{\theta} \rangle \downarrow$ as α drops below unity.



Synthesis of the Experiments

• Shear layer collapse and turbulence and D (particle transport) rise as $\frac{\overline{n}}{\overline{n}_c} \rightarrow 1$.

 \rightarrow Key microphysics of density limit !?

• ZF collapse as $\alpha = \frac{k_{||}^2 v_{th}^2}{|\omega| v_e}$ drops from $\alpha > 1$ to $\alpha < 1$.

 \rightarrow Effect on production

- Degradation in particle confinement at density limit in L-mode is due to breakdown of self-regulation by zonal flow
- Note that β in these experiments is too small for conventional Resistive Ballooning Modes (RBM) explanation.



The Key Questions

- What physics governs shear layer collapse (or maintanance) at high density?
 - \Leftrightarrow 'Inverse process' of familar L \rightarrow H transition !?

i.e.
$$L \rightarrow H$$
: { shear layer \rightarrow barrier
turbulence
Density Limit: strong \leftarrow { shear layer,
turbulence turbulence

→ In particular, what is the fate of shear flow for

hydrodynamic electrons: $k_{\parallel}^2 V_{th}^2 / \omega \nu < 1$?

Simulations !?

• Extensive studies of Hasegawa-Wakatani system

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for k_{\parallel}^2 V_{the}^2 / \omega \nu < 1, > 1 regimes.
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i.e. Numata, et al '07 Gamargo, et al '95 Ghatous and Gurcan '15

- All note weakening or collapse of ordered shear flow in hydrodynamic regime $(k_{\parallel}^2 V_{the}^2 / \omega v < 1)$, which resembles 2D fluid turbulence.
- Physics of collapse left un-addressed, as adiabatic regime $(k_{\parallel}^2 V_{the}^2 \omega / \nu > 1)$ dynamics of primary interest

A Theory of Shear Layer Collapse

A Simple, Generic Model



For neoclassical mean field evolution

 $\rho_i^2 \to \rho_{eff}^2 \to \rho_{\theta i}^2$

Dispersion Relation for $\alpha < 1$ *and* $\alpha > 1$



key: $\alpha < 1 \rightarrow$ drift wave converts to convective cell

Step Back: Zonal Flows Ubiquitous! Why?

• Direct proportionality of wave group velocity and wave energy density flux to Reynolds stress $\leftarrow \rightarrow$ spectral correlation $\langle k_x k_y \rangle$



But NOT for hydro convective cells:

•
$$\omega_r = \left[\frac{|\omega_{*e}|\hat{\alpha}|}{2k_{\perp}^2\rho_s^2}\right]^{1/2} \rightarrow \text{for convective cell of H-W}$$

- $V_{gr} = -\frac{2k_r \rho_s^2}{k_\perp^2 \rho_s^2} \omega_r$ $\leftarrow ?? \rightarrow \langle \tilde{V}_r \tilde{V}_\theta \rangle = -\langle k_r k_\theta \rangle$; direct link broken!
- → Energy flux NOT simply proportional to Momentum flux →
- → Eddy tilting ($\langle k_r k_\theta \rangle$) does <u>not</u> arise as direct consequence of causality
- → ZF generation <u>not</u> 'natural' outcome in hydro regime!
- ➔ Physical picture of shear flow collapse emerges

Reduced Model 🗇 Demonstrate Understanding

- Utilize models for <u>real space</u> structure to address shear layer
 - e.g. { Balmforth, et. al. Ashourvan, P.D. Outgrowth of staircase studies

See also: J. Li, P.D. '2018 (PoP) – saturation for friction \rightarrow 0

- Exploit PV conservation: (PV $\leftarrow \rightarrow$ Potential Vorticity)
 - $q = \ln n ∇^2 φ$ → conserved PV ←→ equivalent to phase space density
- $\begin{array}{ll} \quad \tilde{q} = \tilde{n} \nabla^2 \tilde{\phi} & \langle n \rangle \text{ mean density} \\ \langle \nabla^2 \phi \rangle \text{ mean vorticity} \end{pmatrix} \text{ define mean PV} \\ \mathbf{So} & \langle \tilde{q}^2 \rangle = \varepsilon \text{ fluctuation potential enstrophy} \end{array}$
- Natural description: $\langle n \rangle$, $\langle \nabla^2 \phi \rangle$, $\langle \tilde{q}^2 \rangle = \varepsilon$ ε = fluctuation P.E.

Reduced Model, cont'd

 $\partial_t n = -\partial_x \Gamma_n + D_0 \nabla_x^2 n$

 $\partial_t u = -\partial_r \Pi + \mu_0 \nabla_r^2 u$

$$l_{mix} = \frac{l_0}{\left(1 + \frac{(l_0 \nabla u)^2}{\varepsilon}\right)^{\delta}} \rightarrow l_0$$

N.B.: Encompasses 'predator-prey' model

 $\partial_t \varepsilon + \partial_x \Gamma_{\varepsilon} = -(\Gamma_n - \Pi)(\partial_x n - \partial_x u) - \varepsilon^{\frac{3}{2}} + P$

• Fluxes:

 $\Gamma_n \rightarrow \text{Particle flux } \langle \tilde{V}_x \tilde{n} \rangle$

 $\Pi \rightarrow \text{Vorticity flux } \langle \tilde{V}_x \nabla^2 \tilde{\phi} \rangle = -\partial_x \langle \tilde{V}_x \tilde{V}_y \rangle \text{ (Taylor, 1915)}$

Reynolds Force

 $\Gamma_{\varepsilon} \rightarrow$ turbulence spreading, $\langle \tilde{V}_{\chi} \tilde{\varepsilon} \rangle \rightarrow$ triad interactions

Expression for Transport Fluxes:

 $\rightarrow \Gamma_{\varepsilon} = -l_{mix}^2 \sqrt{\varepsilon} \, \partial_x \varepsilon$

$$\rightarrow \Gamma_{n} = -D \ \partial_{x} n = -\frac{(\hat{\alpha} + |\gamma_{m}|)}{|\omega + i\hat{\alpha}|^{2}} \frac{d \ln n}{dx} \langle \delta v_{x}^{2} \rangle \longrightarrow \text{Diffusive Flux}$$

$$\rightarrow \Pi = -\chi_{y} \ \partial_{x} u + \Pi^{res} \qquad (\text{Physics of vorticity gradient t.b.d.})$$

$$\text{Shear relaxation by turbulent} \qquad \text{viscosity} \qquad \text{Production and acceleration of flow by } \nabla n$$

$$\chi_{y} = \frac{|\gamma_{m}| \langle \delta v_{x}^{2} \rangle}{|\omega|^{2}} \qquad \Pi^{res} = \frac{k_{\theta} \rho_{s} c_{s} \omega_{ci} \hat{\alpha} \left[(\omega^{r})^{2} (\omega^{*} - \omega^{r}) - |\gamma_{m}|^{2} (\omega^{r} + \omega^{*}) - \omega^{*} \hat{\alpha} |\gamma_{m}| \right]}{|\omega|^{2} \times |\omega + i\hat{\alpha}|^{2}} \langle \tilde{\phi}^{2} \rangle$$

Turbulence Spreading

Clear dependence of D, χ_y, Π^{res} on $|\omega|$ and $\hat{\alpha}$

Scaling of transport fluxes with α (adiabaticity parameter)

Plasma Response	Adiabatic (α >>1)	Hydrodynamic (α <<1)	Γ_n, γ
Particle Flux Γ	$\Gamma_{\rm adia} \sim \frac{1}{\alpha}$	$\Gamma_{hydro} \sim \frac{1}{\sqrt{\alpha}}$	elec fror
Turbulent Viscosity χ	$\chi_{adia} \sim \frac{1}{\alpha}$	$\chi_{hydro} \sim rac{1}{\sqrt{lpha}}$	hyd
Residual stress Π ^{res}	$\Pi^{res}_{adia} \sim -\frac{1}{\alpha}$	Π^{res}_{hydro} ~- $\sqrt{\alpha}$	α <
$\frac{\Pi^{\text{res}}}{\chi} = \text{Vorticity Gradient}$	α^0	α^1	pro

 $\Gamma_n, \chi_y \uparrow \text{ and } \Pi^{\text{res}} \downarrow \text{ as the}$ electron response passes from adiabatic ($\alpha > 1$) to hydrodynamic ($\alpha < 1$) $\alpha < 1 \rightarrow \underline{\text{weak flow}}$ production

- Mean vorticity gradient ∇u (i.e. ZF strength) proportional to $\alpha \ll 1$ for convective cells.
- Weak ZF formation for $\alpha \ll 1 \rightarrow$ weak regulation of turbulence and enhancement of particle transport and turbulence.

Some Theoretical Matters

Physics of Vorticity Gradient ?!

- ∇u , not flow shear, is natural flow order parameter
- [Jump in flow shear, over scale l] = [∇u , over scale l]
- Vorticity gradient prevents local alignment of eddy or mode with shear
- $\Pi = 0 \rightarrow \nabla u \sim \Pi^{res} / x_y$
- Standard interpretation: Enhanced 'drift wave elasticity' → ∇u converts turbulence to waves, so reducing mixing.



ZF Collapse $\leftarrow \rightarrow$ PV Conservation and PV Mixing? How reconcile?

Rossby waves:

Ω

 $\mathbf{
u}$

Density

- $PV = \nabla^2 \phi + \beta y$ is conserved from θ_1 to θ_2 .
- Total vorticity $2\vec{\Omega} + \vec{\omega}$ frozen in \rightarrow Change in mean vorticity Ω leads to change in local vorticity $\omega \rightarrow$ Flow generation (Taylor's ID)

Drift waves:

Radius

- In HW, $q = \ln n \nabla^2 \phi = \ln n_0 + h + \tilde{\phi} \nabla^2 \phi$ conserved along the line of density gradient.
- Change in density from position 1 to position
 2→ change in vorticity → Flow generation (Taylor ID)

Quantitatively

- Total PV flux $\Gamma_q = \langle \tilde{v}_x h \rangle \rho_s^2 \langle \tilde{v}_x \nabla^2 \phi \rangle$
- <u>Adiabatic limit $\alpha \gg 1$:</u> +Particle flux and vorticity flux are <u>tightly</u> <u>coupled</u> (both prop. to $1/\alpha$)
- <u>Hydrodynamic limit α ≪ 1 :</u>
 Particle flux proportional to 1/√α.
 Residual vorticity flux proportional to √α.
- PV mixing still possible without ZF formation → <u>Particles</u> carry PV flux
- Branching ratio changes with α !

Some Pragmatic Matters

The Big Picture



A Developing Story

From Linear Zoology to Self-Regulation and its Breakdown



- $\alpha_{MHD} = -\frac{Rq^2d\beta}{dr} \rightarrow \nabla P$ and ballooning drive to explain the phenomenon of density limit.
- Invokes yet another linear instability of RBM.
- What about density limit phenomenon in plasmas with a low β?

State	Electrons	Turbulence Regulation
Base State - L -mode	Adiabatic or Collisionless $\alpha > 1$	Secondary modes (ZFs and GAMs)
<i>H</i> -mode	Irrelevant	Mean ExB shear <i>V</i> Pi/n
Degraded particle confinement (Density Limit)	Hydrodynamic $\alpha < 1$	None - ZF collapse due weak production for $\alpha < 1$

(Hajjar et al., PoP, 2018)

Secondary modes and states of particle confinement

<u>L-mode</u>: Turbulence is *regulated* by shear flows, but not suppressed.

<u>H-mode</u>: *Mean ExB* shear $\leftrightarrow \nabla p_i$ suppresses turbulence and transport.

<u>Approaching Density Limit:</u> High levels of turbulence and particle transport, as shear flows collapse.

i.e. Shear Flow: Density Limit Weak (none) > < L-mode Modest > < H-mode Strong Mean

Partial Conclusions (L-mode)

- 'Density limit' is consequence of particle transport dynamics, edge cooling, etc. secondary.
- Degraded particle confinement <u>shear layer collapse</u>, breakdown of self-regulation; 'Inverse' of L \rightarrow H transition
- Physics: Drop in shear flow production

Key parameter: $k_{\parallel}^2 V_{The}^2 / \omega v_e$ (adiabaticity)

 Penetration of turbulence spreading drives cooling front, related to MARFE etc.

Desperately Seeking Greenwald, and beyond...

- What of current scaling?

- Tokamaks, RFP, Stellarators?

What of the Current Scaling?

- Obvious question: How does shear layer collapse scenario connect to Greenwald scaling $\bar{n} \sim I_p$?
- Key physics: shear/zonal flow response to drive is 'screened' by neoclassical dielectric

i.e. $-\epsilon_{neo} = 1 + 4\pi\rho c^2/B_{\theta}^2$

- $-\rho_{\theta}$ as screening length
- effective ZF inertia lower for larger I_p

Current Scaling, cont'd

• Shear flow drive:

emission from 'drift-mode' interaction

$$\frac{d}{dt} \left[\left\langle \left(\frac{e\phi}{T} \right)^2 \right\rangle_{ZF} \right] \approx \frac{\sum_k |S_{k,q}|^2 \tau_{c_{k,q}}}{|\epsilon_{neo}(q)|^2}$$
neoclassical response

- Production $\leftarrow \rightarrow$ beat drive
- Response (neoclassical)
- Rosenbluth-Hinton '97 et seq

Increasing I_p decreases ρ_{θ} and off-sets weaker ZF drive

$$\begin{pmatrix} e\hat{\phi} \\ T \end{pmatrix}_{ZF} \approx \frac{S_{k,q}}{\left(1 + 1.16 \frac{(q(r))^2}{\epsilon^{1/2}}\right) q_r^2 \rho_i^2}$$
classical neo zonal wave #

Current Scaling, cont'd



- Higher current strengthens ZF shear, for fixed drive
- Can "prop-up" shear layer vs weaker production
- ~ $(1 + 2q^2)\rho_i^2$ for collisional regime

What of other Donuts? Pretzels?

- All devices exhibit edge shear layer in L-mode and many similar fluctuation properties (Carreras, Hidalgo et. al.)
- RFP ~ Cylinder → 'neoclassical' effects ignorable
 But:
- RFP exhibits Greenwald scaling $n \sim I_p$!
- <u>Classical</u> ZF response $\rightarrow \rho_i$, but ρ_i set by current in RFP i.e. $\rho_i = \rho_{\theta i}$
- Stronger ZF shear at higher current, again

What of Stellarator? (Ackn T.-H. Watanabe, Carlos Hidalgo)

- Several attempts to 'translate' Greenwald scaling into stellarator ('magnetic geometry thinking): $B_{\theta} \rightarrow iota$, shear, ...
- Dubious outcomes...
- If ZF screening crucial, better ask: "What length scale appears in Z.F. response for stellarator?"
- Sugama-Watanabe: Principlal correction to classical screening is contribution from helically trapped particle (analysis for LHD).

What of Stellarator?, cont'd

- No obvious length scale emerges
- Need explore collisional regime
- →Begs: Will optimized stellarator have higher

density limit due more robust edge shear

layer?

→Issue remains open

Thoughts for Experiment

Suggestions for Experiment

- Criticality $k_{\parallel}^2 V_{The}^2 / \omega v_e \rightarrow T_e^2 / n_e \text{ trade off}$
- <u>Scale</u> of shear layer collapse? ρ_{θ} ?
- Turbulence spreading penetration depth? influence length
- Perturbative experiments: (J-TEXT, planned)
 - SMBI probe of relaxation (with fluctuations) \rightarrow relaxation time
 - ExB flow drive (Bias) \rightarrow enhance shear layer persistence beyond \bar{n}_g ?
 - RMP \rightarrow accelerate shear layer collapse?
- N.B. Studies of turbulence and transport as $n \rightarrow n_g$, are part of

(important) 'disruption question'.

In Particular:

- Can edge biasing (ala' driven L \rightarrow H) sustain $\bar{n} > \bar{n}_g$ by driving shear layer?
- Is shear layer collapse hysteretic?



• Is shear layer collapse yet another case of a back-transition of transport bifurcation?

What of H-mode?

- H-mode density limit involves back-transition prior to \bar{n}_g , so key HDL problem is high density back-transition (H \rightarrow L)
- *I_{turb}* in SOL can exceed that of pedestal

• Is HDL due

...

- Shear layer or well weakening? How?
- Invasion of pedestal from SOL turbulence
- Coupled pedestal-SOL model under consideration

General Conclusions

- Transport is fundamental to density limit. Cooling, etc.
 drive secondary phenomena.
- Shear layer collapse occurs as transport bifurcation from DW-ZF turbulence to convective cells, approaching density limit.
- Trends of Greenwald scaling follow from neoclassical zonal flow response.

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